Electrokinetic Model Refinement Via a Perturbation Finite-Element Method—From 2-D to 3-D

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A method for increasing the contact area of a grounding system with the earth is the installation of counterpoise wires. Counterpoise wires improve the reliability of overhead power transmission lines. They are conductors buried in the ground parallel to or at an angle to the line conductors. This paper presents an electrokinetic model refinement via a perturbation finite-element method to calculate the grounding resistance of counterpoise wires. The perturbation method is herein developed for refining the electric field distribution in soil starting from simplified models, based on electric field distributions from 2-D models, that evolve towards a 3-D accurate model. The analysis of the distribution of the electric field and of the electric potential around the tower footing allows accurately determining the tower footing resistance. The comparisons between the grounding resistance simulated and calculated analytically of a counterpoise wire are presented.

***Index Terms*—Counterpoise wires, electrokinetic model, grounding resistance, perturbation finite-element method.**

I. Introduction

FOR evaluating the behavior of transmission lines in case of lighting strike, the accurate modeling of tower footing resistance is crucial. In particular, the decrease of the earth resistance observed for high values of the current flowing from the tower to earth has to be accurately considered [1].

High structure footing impedances cause increased voltages and more lightning outages for a given lightning exposure. A complete line design will specify the types and sizes of ground electrodes needed to achieve the required footing impedance. The electrode sizes and shapes will depend on the range of soil conductivities found on installation. In some geographic areas, surveys of apparent ground resistivity have been carried out for radio-frequency broadcast or geological purposes [2].

When a stroke contacts a tower, a portion of the stroke current travels down the tower [2]. The remainder passes out along the overhead ground wires. The initial fractions along these two paths are determined by their relative surge impedances. The tower current flows to earth at the base of the tower through the tower footing impedance. The resultant voltage drop and the magnitude of the voltage wave reflected back up the tower, depend directly on the value of the footing impedance encountered by the current. The tower footing impedance depends on the area of the tower steel (or grounding conductor) in contact with the earth, and on the resistivity of the earth. The latter is not constant; it fluctuates over time and is a function of soil type, moisture content, temperature, current magnitude, and wave shape [2].

Fig. 1 closely represents some towers used in Brazil, especially for 138 kV systems. As the soil dimensions are much bigger than counterpoise wire dimensions, the finite element (FE) of such a 3-D problem becomes computationally expensive. The counterpoise is a conductor buried in the ground parallel to or at an angle to the line conductors. It may be considered a horizontal electrode as compared with the vertical electrode created by a driven ground rod [2]. Common arrangements include one or more radial wires extending out from each tower base; single, or multiple continuous wires from tower to tower; or combinations of radial and continuous wires. The counterpoise may sometimes be augmented with periodic driven rods [2]. Fig. 2 shows the installation of counterpoise wires.

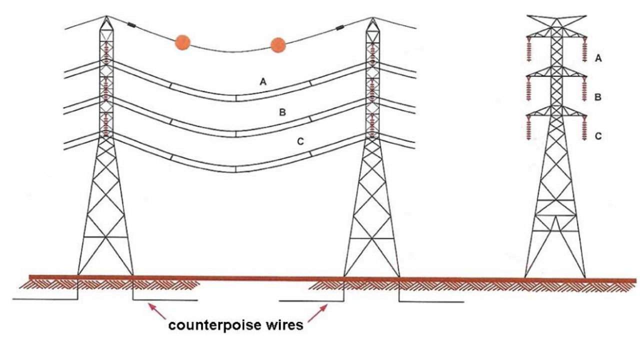

Fig. 1. Transmission tower, aerial cables, tower-footing, and counterpoise wires.

This paper analyzes the behavior of electric field and of electric potential on a counterpoise wire. The purpose of this analysis is to calculate the grounding resistance of this wire.

A perturbation FE method is herein developed for refining the electric field distribution in soil starting from simplified electrokinetic (EK) models, based on electric field distribution from 2-D model, that evolve towards a 3-D more accurate model. It is an extension of the method proposed in [3]–[5]. The developments are performed for the electric scalar potential FE EK formulation, paying special attention to the suitable discretization of the constraints involved in each sub-problem.

The perturbation of FE solutions provides clear advantages in repetitive analyses and helps improving the solution accuracy. It allows to benefit from previous computations instead of starting

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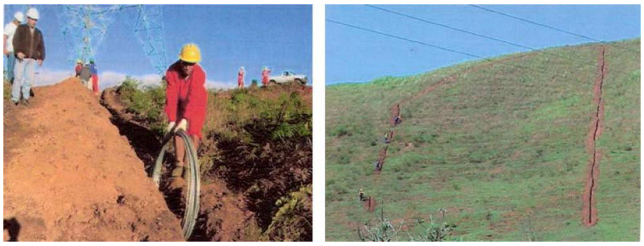

Fig. 2. Installation of counterpoise wires.

a new complete FE solution for any variation of geometrical or physical data. It also allows different problem-adapted meshes and computational efficiency due to the reduced size of each sub-problem.

II. Analytical Equation

The inherent construction of the tower may result in a substantial surface area of tower steel, grillage, and foundation reinforcing cages in contact with the earth. A method for increasing the contact area of a grounding system with the earth is the installation of a counterpoise [2] (see Figs. 1 and 2).

As the current reaches more of the conductor, it effectively uses more of the contact area with the earth. The impedance thus decreases with time and reaches a steady-state value when the current is distributed through the entire length. The steady-state contact resistance may be calculated as [6]

$$R = \frac{\rho}{\pi L}\left[\ln\frac{2L}{\sqrt{2rh}} - 1\right] \tag{1}$$

where ρ is the resistivity of the earth in $(\Omega \cdot m), r$ is the wire radius in (m), h is the wire depth in (m), L is the counterpoise length (m), and $L \gg h$.

The steady-state contact resistance is not greatly influenced either by r or h. Traditional burial depth for a counterpoise is from about 0.5 m to 1 m. For a 20 mm diameter, 100 m long counterpoise, increasing the burial depth from 0.5 m to 1.5 m would decrease resistance by less than 9% [2]. The choice of a thin, wide strap cross section, rather than a large circular wire, may reduce inductive effects by as much as 15% and may increase exposed surface area at the same time. Several short wires, arranged radially, may be more effective than a single long wire even if the total length and contact resistance of both are the same [2].

III. Reference and Modified Problems

A. Canonical Electrokinetic Problem

A canonical EK problem p is defined in a bounded domain Ω_p, with boundary $\partial\Omega_p = \Gamma_{e,p} \cup \Gamma_{j,p}$, of the 2-D or 3-D Euclidean space. Its equations and material relation in Ω_p, and boundary conditions (BCs) and interface conditions (ICs) on $\partial\Omega_p$ are

$$\operatorname{curl} \boldsymbol{e}_p = 0, \quad \operatorname{div} \boldsymbol{j}_p = 0, \quad \boldsymbol{j}_p = \sigma \boldsymbol{e}_p \tag{2a-b-c}$$

$$\boldsymbol{n} \times \boldsymbol{e}|_{\Gamma_{e,p}} = 0, \quad \boldsymbol{n} \cdot \boldsymbol{j}|_{\Gamma_{j,p}} = 0, \tag{2d-e}$$

$$[\boldsymbol{n} \times \boldsymbol{e}_p]_\gamma = \boldsymbol{e}_{f,p}, \quad [\boldsymbol{n} \cdot \boldsymbol{j}_p]_\gamma = \boldsymbol{j}_{f,p} \tag{2f-g}$$

where $\boldsymbol{e}$ is the electric field, $\boldsymbol{j}$ is the electric current density, σ is the electric conductivity, and $\boldsymbol{n}$ is the unit normal exterior to Ω_p. According to (2a), the electric field can be expressed in terms of an electric scalar potential v_p, i.e., $\boldsymbol{e}_p = -\operatorname{grad} v_p$. The BC (2d) defines a constant scalar potential on each non-connected part of $\Gamma_{e,p}$. It is applied on the boundary $\Gamma_{c,p}$ of each perfect conductor $\Omega_{c,p}$ and on the possible infinity boundary Γ_∞ of Ω_p. At the discrete level, independent meshes are used for all problems p.

The notation $[\,\cdot\,] = \cdot|_{\gamma+} - \cdot|_{\gamma-}$ refers to the discontinuity of a quantity through any interface γ (of both sides γ^+ and γ^-; the region in between is exterior to Ω_p) [see Fig. 4(a)], which is allowed to be non-zero; the associated surface fields $\boldsymbol{e}_{f,p}$ and $\boldsymbol{j}_{f,p}$ are usually unknown, i.e., parts of the solution [5]. It is intended to solve successive problems, the solutions of which being added to get the solution of a complete problem [5].

The portions of a 3-D structure satisfying a translational or rotational symmetry can be first studied via 2-D models. For a portion Ω_q, this consists in neglecting some end effects, zeroing either $\boldsymbol{n} \times \boldsymbol{e}$ or $\boldsymbol{n} \cdot \boldsymbol{j}$ on the interfaces γ_q separating it from another portion. Furthermore, if the field is chosen to be zero out of this portion, a discontinuity of the remaining non-zero trace is then voluntarily defined through γ_q. The domain Ω_q can be thus reduced to its 2-D cross section. A 3-D problem p has then to correct this assumption in a certain neighborhood Ω_p on both sides of the interface $\gamma = \gamma_p = \gamma_q$ (γ_p and γ_q only differ at the discrete level by their meshes) via the ICs (2f-g). Their sources $\boldsymbol{e}_{f,p}$ and $\boldsymbol{j}_{f,p}$ are obtained from the 2-D solution q. They express opposite discontinuities to recover the actual continuities of the total fields, i.e.,

$$\boldsymbol{e}_{f,p} = -\boldsymbol{e}_{f,q}, \quad \boldsymbol{j}_{f,p} = -\boldsymbol{j}_{f,q}. \tag{3a-b}$$

Given that each solution is calculated on a different mesh, mesh-to-mesh projections of solutions are required [5]. This is a key point of the method for ensuring continuity.

B. Perturbation Problems

A modification of an initial problem $p = 1$ due to a change of conductivity and/or an addition of sources in some sub-regions leads to the perturbation of the field quantity. Both large and small perturbations can be accounted for, e.g., adding new materials, new regions, etc.

In this work, the perturbing regions will then be additional regions that influence the initial electric field distribution.

The perturbation FE method consists thus in determining the solution of P successive sub-problems $p = 1, \ldots, P$, the addition of which being the solution of the complete problem. The complete solution is then [6]

$$v = \sum_{p=1}^{P} v_p, \quad \boldsymbol{e} = \sum_{p=1}^{P} \boldsymbol{e}_p, \quad \boldsymbol{j} = \sum_{p=1}^{P} \boldsymbol{j}_p. \tag{4a-b-c}$$

As each sub-problem is generally perturbed by all the others, each solution v_p has to be calculated as a series of corrections, i.e., [6]

$$v_p = v_{p,1} + v_{p,2} + \cdots. \tag{5}$$

The calculation of the correction $v_{p,i}$ in a problem p, i is kept on till convergence up to a desired accuracy. Each correction $v_{p,i}$ must account for the influence of all the previous corrections $v_{q,j}$ of the other sub-problems, with $q = 1, \ldots, p-1, j = i$ and $q = p+1, \ldots, P, j = i-1$. Further, initial solutions $v_{p,0}$ are set to zero [6]. The iterative process is justified by the fact that a correction can become a significant source for any of its source problems, which is proper to large perturbation problems. In addition to the iterations between sub-problems, classical inter-problem iterations are needed in nonlinear analyses.

In this work, combinations of 2-D and 3-D models are considered to accurately calculate the electric field in the vicinity of the end of the wire.

C. Canonical Problem in a Weak Form

The electric scalar potential formulation of each EK problem p (2) is given by

$$(\sigma_p \operatorname{grad} v_p, \operatorname{grad} v')_{\Omega_p} + \langle \boldsymbol{n} \cdot \boldsymbol{j}_p, v' \rangle_{\Gamma_p} = 0, \quad \forall v' \in F_v(\Omega_p) \qquad (6)$$

where $F_v\ (\Omega_p)$ is the function space defined on Ω_p and containing the basis functions for v_p as well as for the test function v' [6]. At the discrete level, $F_v\ (\Omega_p)$ is approximated with nodal FEs. $(\cdot,\cdot)_\Omega$ and $\langle \cdot,\cdot \rangle_\Gamma$ respectively denote a volume integral in Ω and a surface integral on Γ of the product of their vector or scalar field arguments.

For the typical 2-D-3-D problem splitting considered here, the 3-D problem p has to correct the 2-D solution q via IC (2f) with (3a); the source (3b) in (2g) is zero. This is done by fixing a discontinuity of v_p through the interface γ equal to the opposite of the 2-D solution v_q. This source solution has to be projected from its supporting 2-D mesh q onto the interface γ of the 3-D mesh p, via the Galerkin projection of its gradient [5], i.e.,

$$\langle \operatorname{grad} v_{q,p-\mathrm{proj}}, \operatorname{grad} v' \rangle_\gamma = \langle \operatorname{grad} v_q, \operatorname{grad} v' \rangle_\gamma, \quad \forall v' \in F_p(\gamma) \qquad (7)$$

of which the solution $v_{q,p-\mathrm{proj}}$ is the projection of v_q on mesh p.

A global basis function is associated to each non-connected portion of $\Gamma_{e,p}$. It equals one on this portion and varies continuously in Ω_p up to zero on the other portions [6]. At the discrete level, such a function can be defined as the sum of the nodal FE basis functions of the nodes of the boundary portion. Such a function, when applied as test function v' in (6), allows to determine the current flowing from the associated boundary.

Resistances are then straightforwardly calculated from the values of voltages and currents [6], via successive corrections. Formulation (6) is valid for any correction $v_{p,i}$ of (5) involved in the iterative process.

IV. Results

The experimental example considered for validation of the proposed approach is a 10 m length counterpoise wire of diameter 0.1 m. It is embedded in the soil at a depth of 1 m. The resistivity of the soil is 300 Ω m.

Figs. 3 and 4 show the 2-D and 3-D calculation domains with their meshes.

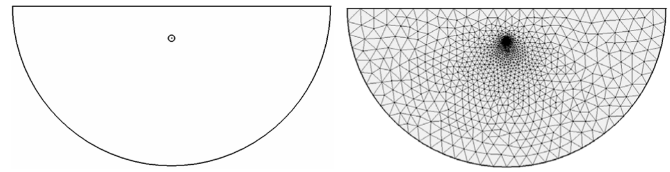

Fig. 3. The 2-D studied domain (*left*) and its mesh (*right*).

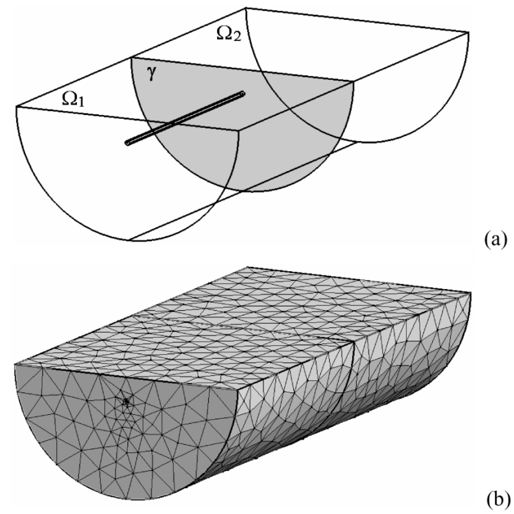

Fig. 4. (a) The 3-D studied domain and (b) its mesh.

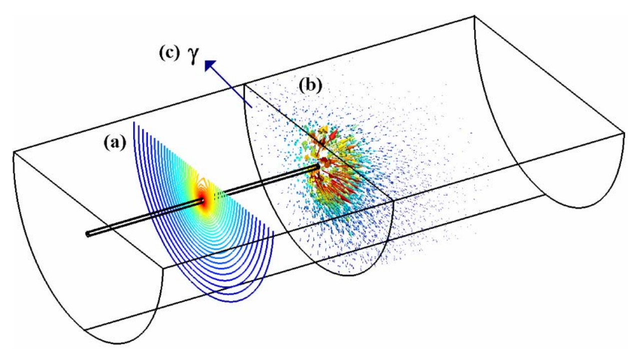

Fig. 5. (a) Electric scalar potential: solution of the 2-D model in XY plane, (b) Electric field: solution of the 3-D model—part of the 3-D correction in the region of the counterpoise wire extremity and (c) Interface γ of the studied domain.

The cross section of the wire and the soil in the XY plane initially defines a 2-D model (Fig. 3), of which the solution is shown in Fig. 5. This 2-D solution is considered to be invariant in the Z direction up to a certain distance. Beyond this distance, the electric field is chosen to be zero, which results in a particular IC to be further corrected. This solution then serves as source in a local 3-D model, fixing a discontinuity of the 3-D electric scalar potential equal to the opposite of the 2-D potential, for a perturbation problem allowing electric leakage flux in 3-D. The 3-D model allows accurately calculating the electric field in the vicinity of the end of the wire (Fig. 5), with its own adapted mesh. This way, a better accuracy is obtained for the grounding footing resistance.

Fig. 6 shows the distributions of the electric scalar potential for the 2-D and 3-D sub-problems.

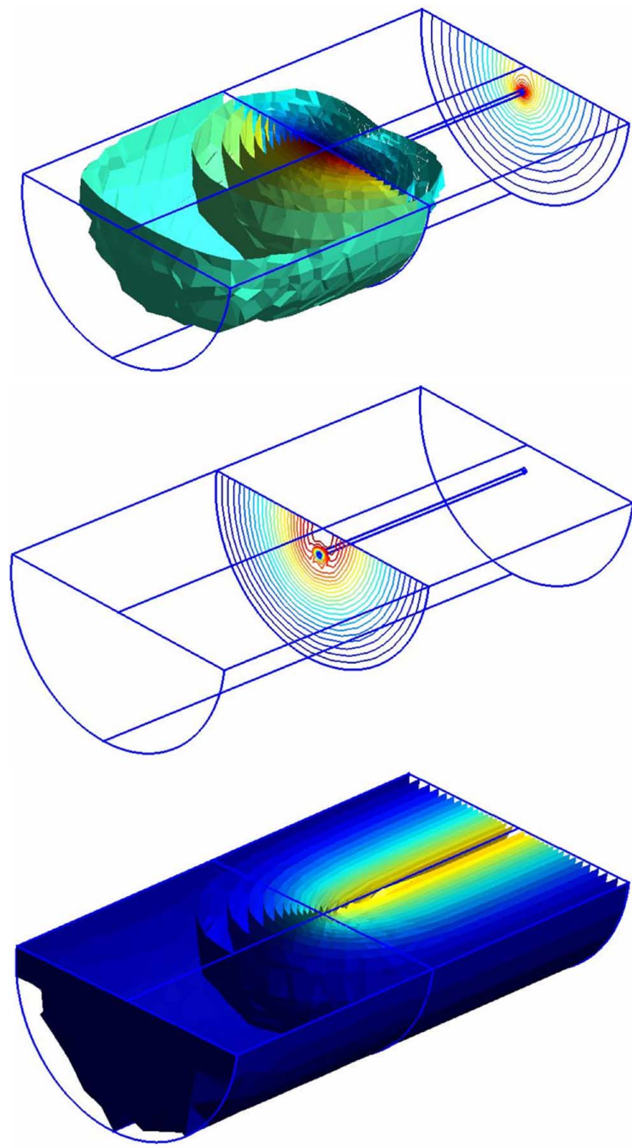

Fig. 6. Electric scalar potential: (a) solution of the 2-D model in XY plane and solution of the 3-D model in the neighborhood of the interface γ; (b) perturbation solution in the interface γ; (c) complete solution combining 2-D and 3-D solutions in the whole studied domain.

TABLE I
GROUNDING RESISTANCE OF A COUNTERPOISE WIRE

	Grounding Resistance (Ω)	*Difference* (%)
Analytical Equation	30.05	0
2-D FEM	26.70	11.15
Perturbation FEM	31.40	4.50
3-D FEM	31.20	3.83

The comparisons between the grounding resistance simulated and calculated analytically are performed and the results are presented in Table I.

The 2-D result can be seen as a part of the 3-D result, to which the 3-D correction problem adds the actual contribution of the end effects. The differences shown in Table I are calculated in relation to the analytical result.

The perturbation FE method shows a good agreement with the analytical one. It offers the advantage of being applicable to more general configurations. In comparison with full 3-D analyses, it allows a reduced computational cost especially in parameterized analyses.

The method can be generalized to any number of interfaces, allowing the connection of several straight portions of wires. In addition, it can be coupled to the correction procedure developed in [6], combining axisymmetrical problems.

V. CONCLUSION

This paper analyses the distribution of electric field and electric potential originated from a counterpoise wire. The purpose of the analysis is to calculate the grounding resistance of this wire. An EK model refinement is done via a perturbation FE method from 2-D to 3-D. The perturbation FE method is herein developed for refining the electric field distribution in soil starting from simplified models, based on electric field distributions from 2-D models, that evolve towards an accurate 3-D model.

The 3-D correction model allows accurately calculating the electric field in the vicinity of the end of the wire, with its own adapted mesh. This way, it gains in accuracy for the benefit of a more accurate evaluation of the grounding resistance. The comparisons between the grounding resistance simulated and calculated analytically were performed and the results presented good agreement. Although promising results can be observed, the modeling of tower footing resistance remains under investigation.

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